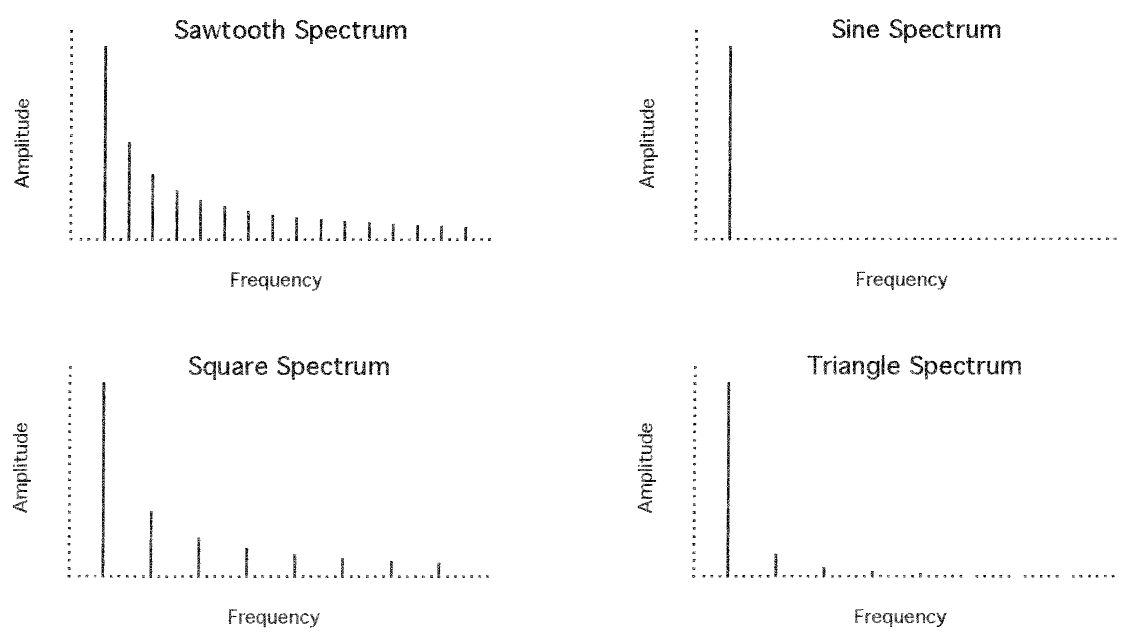
Signal Waveform/Spectrum

A *periodic* signal has a *waveform*–a shape that *repeats*. Each repetition requires the same *period* of time: one *cycle*–a single iteration of the waveform. *Frequency* is the number of times a periodic waveform repeats in one second. Frequency is measured in Hertz (Hz), also known as cycles per second (cps). A waveform with a frequency of 1000 Hertz (Hz) repeats 1000 times per second. *Period* is measured in time, and is the *reciprocal* of frequency. A waveform with frequency of 1000 Hertz has a period of 1/1000 second, or 1 millisecond.

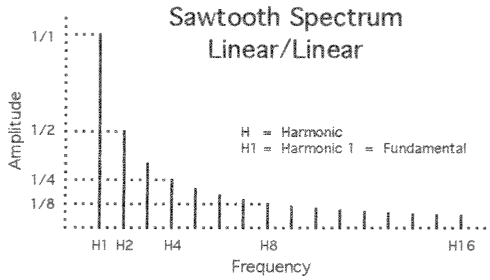
In musical terms, audible periodic waveforms (waveshapes, waves) are *pitched. The* relationship between pitch and frequency is not perfect, but the waveform with a higher frequency is usually heard as a higher pitch. All periodic signals are not within the range of human hearing, the *audible window* (20 Hz to 20,000 Hz). Frequencies below 20 Hertz are known as *subsonic* or *infrasonic.* Frequencies above our upper hearing limit (20 kHz, or 20 kiloHertz) are known as *supersonic* or *ultrasonic.* In music synthesis, a signal is defined as an "audio" signal solely by how it *functions*, when it is connected to an *audio input,* not by *any* of its signal characteristics such as frequency.

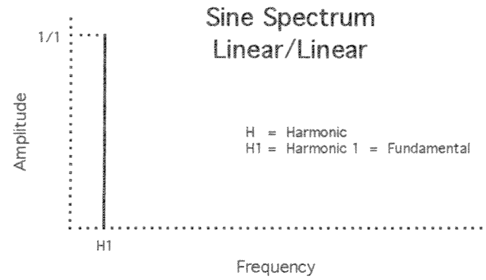
A steady tone made by a musical instrument appears on an *oscilloscope* as a distinctive waveform. This shape, or waveform is a single line on the oscilloscope, which can dynamically graph sound pressure level (SPL) or voltage as it changes in time. Since waveform *is* a representation of sound in time, this depiction is known as the *time domain.* Time is shown along the horizontal axis, and the amplitude, or size of the waveform is shown on the vertical axis. *If the waveform is audible,* then amplitude is perceived as loudness. In the time domain we identify classic waveforms such as the *sawtooth, square, triangle, and sine* because each shape suggests its name. (*Sine* in Latin means *without,* implying, in this context, *without harmonics*). Dynamic waveforms are difficult to show in print, but a static waveform may be depicted as a few cycles of the *waveform* in the *time domain:*

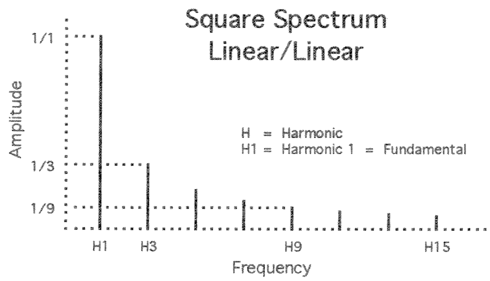
These waveforms are called *geometric* because they can be generated mathematically by summing *trigonometric* functions that represent individual *partials*–each partial having a specific frequency and amplitude. These are the waveforms produced by a typical VCO or fu*nction generator.* The *partials* of a waveform can be viewed on a *spectrum analyzer,* and shown in the *frequency domain:*

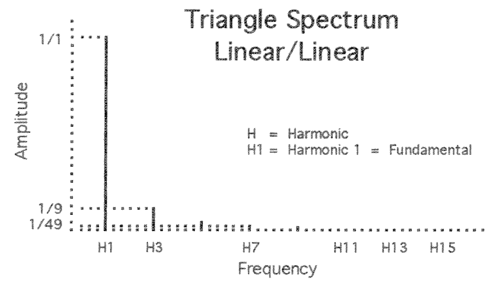


A *line spectrum* comprises vertical line(s), each of which represents a partial, or sine wave. The height of each partial represents its amplitude relative to some *full scale* amplitude, e.g. 1 volt. The horizontal axis represents frequency, lows on left. The geometric waveforms are shown with greater detail below:



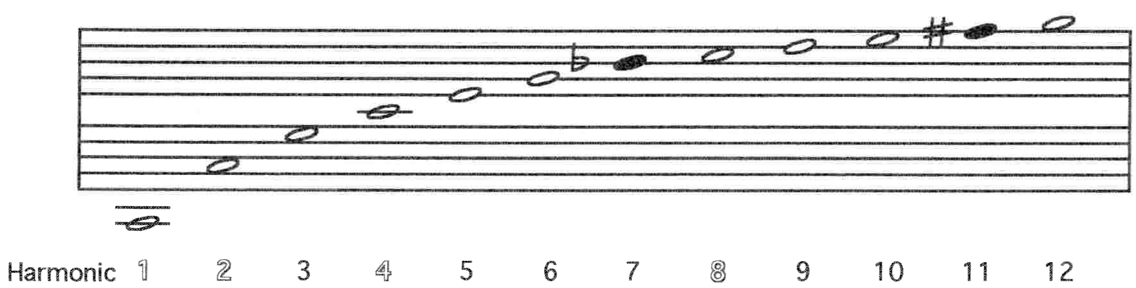






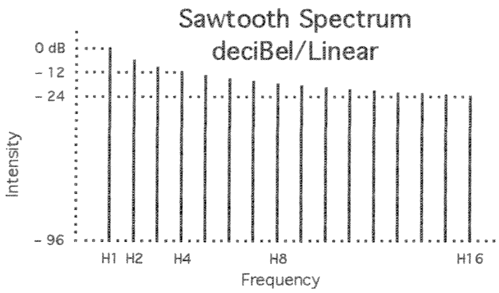
These *spectra* depict frequency *linearly:* equal frequency increments appear at equal distances on the horizontal axis (see sawtooth spectrum). *Harmonics* are spaced at equal frequency increments of the fundamental (H1) from each other. With a *fundamental* of 100 Hz, successive harmonics H2, H3, H4 are frequencies 200, 300, 400 Hz, *equidistant* from each other. Musical intervals are *not* based on linear frequency relationships–intervals are *ratiometric.* An octave is a *doubling* of frequency, a 2:1 ratio. Higher octaves require successive *multiplications* by this factor of 2:1. Given a fundamental of 100 Hz, successive *octaves* H2, H4, H8, H16 have frequencies 200, 400, 800, 1600 Hz, and *do not* appear *equidistant* on the horizontal axis in this *linear* representation of frequency. A musically notated representation of a Harmonic Series shows musical *intervals* (e.g. octaves) with equidistant spacing: musical notation is a *logarithmic* representation of frequency.

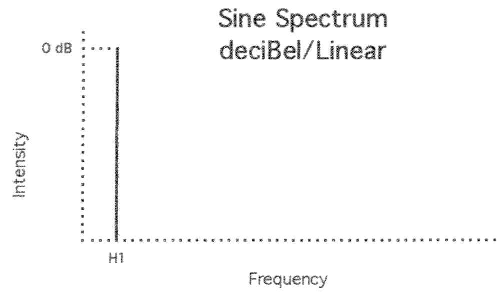
Harmonic Series (Fundamental C2 = Harmonic1)

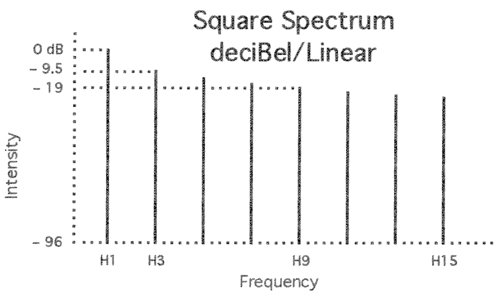


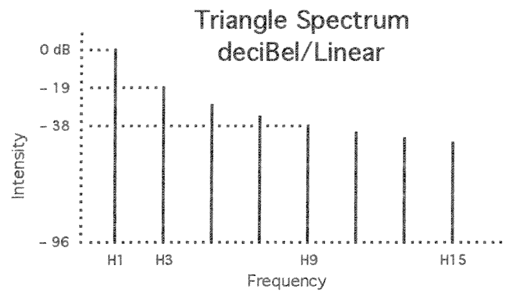
The *spectra* above depict amplitude *linearly*, with equal increments of amplitude (typically measured in volts) represented spatially by equal distances on the vertical axis. For example, let's imagine the *sawtooth* spectrum's fundamental has a *full scale* amplitude of 1 volt. The amplitude of harmonic 2 is 1/2 of *full scale*–or 1/2 volt, and so forth. That is, the amplitude of any harmonic in the sawtooth is the *reciprocal* of its harmonic number. Therefore, the amplitude of harmonic number 3 is 1/3 of the *fundamental's* amplitude. The square has reciprocal amplitudes like the sawtooth, but has *odd* partials only. The triangle has odd partials with *reciprocal squared* amplitude relationships. The amplitude of triangle waveform harmonic number 3 is (1/3 times 1/3), or 1/9 that of full scale (1/1) amplitude.

A linear representation of amplitude is like a cookbook, with an apparent amplitude *factor* for each partial in the waveform's spectrum. However, a linear amplitude representation visually exaggerates the actual progressive power loss, or reduction of intensities of upper harmonics. In the triangle wave spectrum shown *above*, it appears that upper partials are almost nonexistent, but our ears inform us otherwise! Since power and intensity are measured in terms of the deciBel (dB), a dB representation of amplitude better depicts relative intensities, or powers of the partials in a line spectrum:



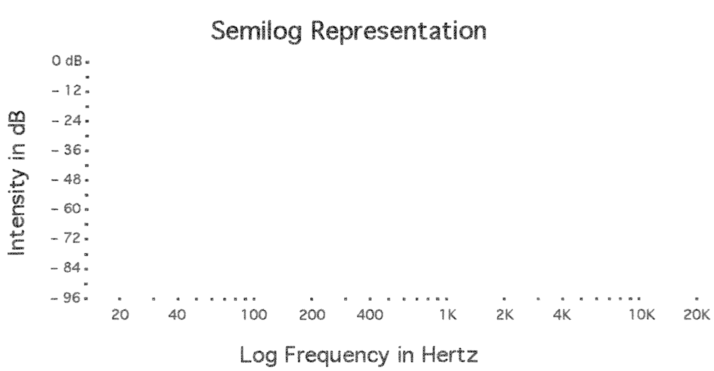






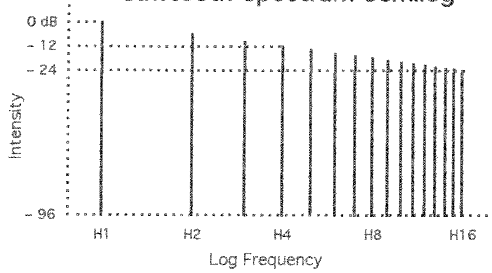
Full scale *intensity*, or *power* is 0 dB (zero deciBels) above. Zero dB doesn't represent "zero power." (On the VU meter in the recording studio, recall that 0 dB is a *large* signal in terms of audio, about 0.775 volts!) Zero dB is a reference and can be put where we *agree* to place it. Power *loss* for each upper harmonic above is expressed in *negative* dB *relative* to this 0 dB reference. In the sawtooth spectrum above, the change of *intensity* from H1 to H2 is – 6dB. Why? Because the H1 to H2 *amplitude* change is 1/2 (see the linear amplitude representation), and there is a *square* relationship between *amplitude* and *intensity*. So, a halving (1/2) of amplitude is a quartering (1/2 times 1/2), or 1/4 of intensity, or power due to this *square* relationship. A quartering of *power* is approximately – 6 dB.

Compare the triangle spectrum above with the previous linear representation of its amplitude! (The "truth" about what we actually *hear* may lie somewhere between the two different representations). Above, with the dB representation, now the power of the upper partials is at least apparent! Given careful inspection, the sawtooth spectrum immediately above reveals a power loss of 6 dB per octave (– 6 dB/oct) for upper partials relative to the fundamental. This would be more obvious *visually* if the *interval* of the octave were represented spatially as an *equal* distance (as our ears actually perceive frequency!) To achieve this, a logarithmic, or "log" *frequency* representation would be necessary:

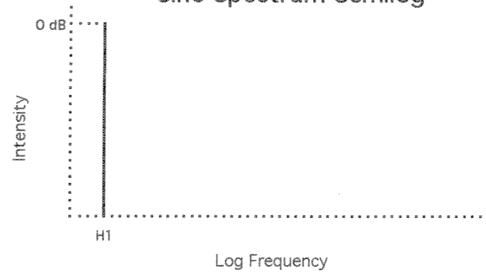


The graphic above depicts frequency *logarithmically.* Equal frequency *ratios* (musical intervals) are represented as equal distances on the horizontal axis. In the sawtooth spectrum below, octaves H1, H2, H4, H8 are equal frequency *ratios,* shown equidistant, as on a piano keyboard. A keyboard is a *logarithmic* representation of *frequency,* and a *linear* representation of *interval!* The *logarithm* of each number is depicted spatially in a logarithmic representation, not the number *per se.* A *logarithm* is an exponent, a power to which some base must be raised to yield a particular number. The graphic above (and spectra below) are *semilog:* linear vertical axis, and logarithmic, or "log" horizontal axis.

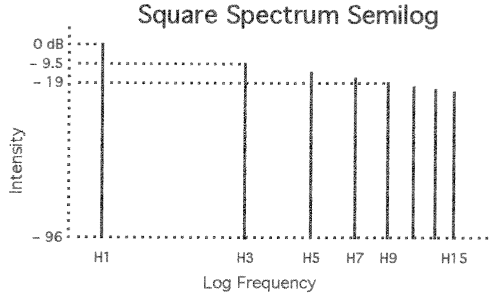
Sawtooth Spectrum Semilog



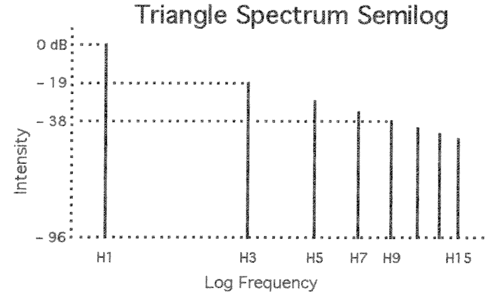
Sine Spectrum Semilog



Square Spectrum Semilog

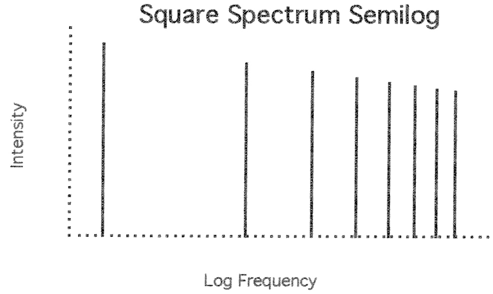


Triangle Spectrum Semilog

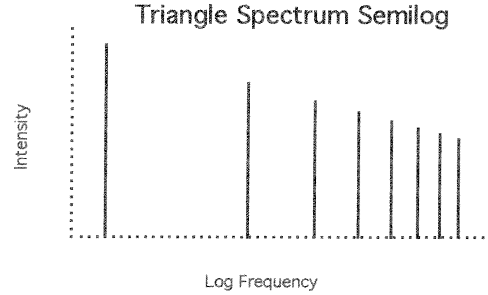


The relative intensities of harmonics in sawtooth and square waveforms *decrease* at a slope of about – 6 dB/octave as frequency *increases.* The intensities of triangle wave harmonics *decrease* at a slope of – 12 dB/octave as frequency *increases*. Slope is apparent when graphics are simplified:

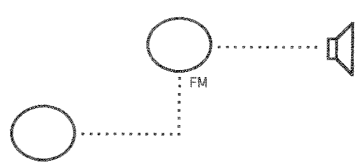
Square Spectrum Semilog



Triangle Spectrum Semilog



To illustrate signal *waveform* sonically, connect the patch shown opposite, tune the *control* VCO to about 1 Hz, and select VCO waveforms at will. Waveform of the audio VCO is not critical; it is only the carrier of the wave*shape* produced by the control VCO:



The illustrate signal *spectrum* sonically, connect the patch shown opposite, tune the resonance of the LP VCF high, and move the filter *cutoff frequency* at will to hear each harmonic individually. Select and audition the *spectrum* for each *audio* VCO waveform: